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THEORETICAL ANALYSIS OF OSCILLATIONS OF A TOWED CABLE

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SUMMARY

Previous analyses of the stability of towed bodies stabilized by fins have generally assumed that the towing cable remained steady. These theories failed to predict the violent motion of these bodies which has been observed to occur in practice above a certain airspeed. This motion involves oscillations of the cable. A theoretical investigation was therefore made of the stability of oscillations of the cable.

This theoretical analysis indicates that oscillations traveling downwind along the cable are amplified by aerodynamic forces when the airspeed is greater than the speed of propagation of waves along the cable. The oscillations are slightly damped when the airspeed is less than the speed of wave propagation. Waves traveling upwind along the cable are always damped.

This theory provides a possible explanation for the violent motions of towed airspeed heads which appear above a certain speed. These oscillations are attributed to cable oscillations which originate near the airplane and are amplified by aerodynamic forces as they travel down the cable. Means are discussed for increasing the speed at which these oscillations become violent.

INTRODUCTION

A previous analysis of the stability of a body stabilized by fins and suspended from an airplane (reference 1) indicated that such a device would be stable when towed at a considerable distance behind an airplane because of the damping action of drag forces on the cable. The type of motion considered in that analysis was a long-period motion in which the cable was assumed to remain steady. In practice such bodies, which are used as towed airspeed heads, generally remain stable in flight up to a certain speed but above this speed violent short-period oscillations have been observed involving pitching and vertical motions of the body and oscillations of the cable. These oscillations have frequently resulted in the body's breaking loose from the cable even though the cable attachment was designed to withstand a load of 25 times the weight of the body. In making airspeed calibrations with towed airspeed heads, the speed at which instability has occurred has been found to depend on the type of airplane from which the body is lowered, the flap and power settings, and the point on the airplane from which the body is suspended. It has been thought that this violent motion was caused by the action of the turbulent wake of the airplane on the cable, inasmuch as the cable trails almost straight back from the airplane at high airplane speeds. The violence of the motions of the towed body has, however, been difficult to explain on this basis alone because the oscillation usually appears to be small near the airplane. The possibility was therefore investigated that oscillations of the cable originating near the airplane might be amplified by the action of air forces as they were propagated down the cable.

SYMBOLS

A	undetermined function of x
a	velocity of wave propagation along cable in vacuum $\left(\sqrt{T/\mu}\right)$
C	constant
c	abbreviation used in trigonometric derivation (α_o)
C_D	drag coefficient of cable when perpendicular to air stream
d	abbreviation used in trigonometric derivation $\left(\frac{\partial y}{\partial x} + \frac{\cos \alpha_o}{V} \frac{\partial y}{\partial t}\right)$
D	diameter of cable
F	force
e	base of natural logarithms
g	acceleration due to gravity

$i = \sqrt{-1}$

$K = C_D \frac{\rho}{2} D \sin 2\alpha_o$

P	period of oscillation of cable
S	equivalent flat-plate area
T	tension in cable
t	time

u	real part of root of equation (see formula (12))
U	velocity of propagation of wave along cable
v	imaginary part of root of equation (see formula (13))
V	airspeed
x	distance along cable
y	distance perpendicular to cable
α	angle of attack of cable

$$\gamma = \cos \alpha_o + \frac{2 \sin^3\alpha_o}{\sin 2\alpha_o}$$

$$\eta = \frac{KV_o^2}{2T}$$

λ	root of equation (u + iv)
μ	mass of cable per unit length
Λ	wave length of oscillation of cable
ρ	air density
ω	circular frequency of forced oscillation of cable

Subscript:

o	equilibrium value

ANALYSIS

When a disturbance is introduced at the end of a long cable which is in tension, the wave form of the disturbance remains unchanged as it travels down the cable provided that aerodynamic forces acting on the cable are neglected. A physical picture of the effect of aerodynamic forces on a cable in an air stream may be obtained from figure 1. In this figure a wave is shown at successive instants traveling down a cable in the same direction as the air velocity. This figure shows that if the airspeed is considerably greater than the speed of the wave the aerodynamic force on each element of the cable acts in the same direction as the transverse (in this case, vertical) motion of the element. The air forces would therefore be expected to feed energy into the oscillation and thereby to increase the amplitude of the oscillation as it travels down the cable. The conclusion may be reached from similar considerations that if the airspeed is the same as the speed of propagation of the wave no aerodynamic effects will be present, because in this case there is no relative transverse motion between the air and the elements of the cable. If the airspeed is less than the speed of propagation of the wave, the air forces will have a slight damping effect.

A more detailed analysis is necessary to determine the rate of damping or amplification of the waves as a function of airspeed and also to find whether the aerodynamic forces affect the speed at which the wave travels down the cable. For this purpose the differential equation of motion of the cable will be set up. The method of analysis is similar to the treatment given in reference 2 of the propagation of transverse waves along a cable where the motion of elements of the cable is resisted by viscous damping forces.

The method of calculating the forces on an element dx of a cable undergoing a small transverse oscillation is illustrated in figure 2. In its equilibrium position, the cable lies along the x-axis and makes an angle α_o with air stream of velocity V_o. The forces acting normal to the cable are now determined. The transverse force on an element of the cable due to tension in the cable is

$$dF_{(tension)} = -T \frac{\partial y}{\partial x} + T \left(\frac{\partial y}{\partial x} + \frac{\partial}{\partial x} \frac{\partial y}{\partial x} dx\right) = T \frac{\partial^2 y}{\partial x^2} dx \tag{1}$$

The gravitational force normal to an element of the cable, if the air stream is assumed to be horizontal, is

$$dF_{(gravity)} = -\mu g\ dx \cos \alpha_o \tag{2}$$

The aerodynamic force is assumed to act normal to the cable element and is given by the formula:

$$dF_{(aerodynamic)} = C_D \frac{\rho}{2} V^2 D\ dx \sin^2\alpha \tag{3}$$

The variation of force with $\sin^2\alpha$ is in accordance with the concept that the force on the cable is due to the component of relative velocity normal to the cable. Measurements of the air forces on cables have shown good agreement with this law (reference 3). This formula holds only in the positive range of angles of attack. The force should vary as $-\sin^2\alpha$ for negative values of α. The neglect of the reversal in force at negative values of α will not lead to any difficulty inasmuch as the cable will be assumed to have an initial positive angle of attack so that the angle of attack will never become negative when the cable performs small oscillations.

The angle of attack of the cable element is

$$\alpha = \alpha_o - \frac{\partial y}{\partial x} - \frac{\cos \alpha_o}{V}\frac{\partial y}{\partial t}$$

In this expression, α_o may be a large angle but the remaining terms are assumed to be small. Hence, the expression for α has the form

$$\alpha = c - d$$

where d is a small quantity. The expression $\sin^2\alpha$ is given by the trigonometric formula:

$$\sin^2\alpha = \sin^2(c - d) = \frac{1}{2} - \frac{\cos 2c \cos 2d + \sin 2c \sin 2d}{2}$$

Since d is small, $\cos 2d = 1$ and $\sin 2d = 2d$
Hence,

$$\sin^2\alpha = \frac{1}{2} - \frac{\cos 2c + 2d \sin 2c}{2}$$

$$= \sin^2 c - d \sin 2c$$

The values for c and d may be placed in this expression as follows:

$$\sin^2\alpha = \sin^2\alpha_o - \left(\frac{\partial y}{\partial x} + \frac{\cos \alpha_o}{V}\frac{\partial y}{\partial t}\right)\sin 2\alpha_o \tag{4}$$

The relative velocity of the cable element and the air V is given by the expression:

$$V = V_o - \frac{\partial y}{\partial t}\sin \alpha_o$$

If higher-order terms are neglected,

$$V^2 = V_o^2 - 2V_o \frac{\partial y}{\partial t}\sin \alpha_o \tag{5}$$

The aerodynamic force on the cable element, obtained by substituting the value for $\sin^2\alpha$ from formula (4) and the value for V^2 from formula (5) in formula (3), is as follows:

$$dF_{(aerodynamic)} = C_D \frac{\rho}{2} D\left(V_o^2 - 2V_o \frac{\partial y}{\partial t} \sin \alpha_o\right)\left[\sin^2\alpha_o - \left(\frac{\partial y}{\partial x} + \frac{\cos \alpha_o}{V_o - \frac{\partial y}{\partial t} \sin \alpha_o} \frac{\partial y}{\partial t}\right)\sin 2\alpha_o\right] dx$$

If higher-order terms are neglected,

$$dF_{(aerodynamic)} = C_D \frac{\rho}{2} DV_o^2 \; dx \sin^2\alpha_o - \frac{\partial y}{\partial x} C_D \frac{\rho}{2} DV_o^2 \; dx \sin 2\alpha_o - \frac{\partial y}{\partial t} C_D \frac{\rho}{2} DV_o \left(\sin 2\alpha_o \cos \alpha_o + 2 \sin^3\alpha_o\right) dx \tag{6}$$

For brevity, let

$$K = C_D \frac{\rho}{2} D \sin 2\alpha_o$$

The equation of motion of the cable element, obtained by equating the force due to tension, equation (1), the gravitational force, equation (2), and the aerodynamic force, equation (6), to the inertia force on the element, is

$$T \frac{\partial^2 y}{\partial x^2} dx - \mu g \; dx \cos \alpha_o + C_D \frac{\rho}{2} DV_o^2 \; dx \sin^2\alpha_o - \frac{\partial y}{\partial x} KV_o^2 dx - \frac{\partial y}{\partial t} KV_o \left(\cos \alpha_o + \frac{2 \sin^3\alpha_o}{\sin 2\alpha_o}\right) dx = \mu \frac{\partial^2 y}{\partial t^2} dx \tag{7}$$

The term dx may be canceled. The resulting equation is one of the differential equations governing the shape and motion of the cable. Another equation, obtained from the equilibrium of forces acting along the cable, would be required to obtain a solution for the cable shape. This additional equation will not be used in the present analysis, however, because this analysis is concerned only with the motion of the cable, which involves deviations from its equilibrium condition. For the same reason, the steady-state terms will be dropped from equation (7). When the cable is in equilibrium, the steady aerodynamic force, given by the third term, may be balanced partly by gravity or partly by a gradual curvature of the cable, as represented by a constant portion of the first term. This curvature will cause a variation of angle of attack along the cable. Also, the tension may vary along the cable. It will be assumed, however, that the cable in equilibrium is straight and that the angle of attack α_o and the tension are constant. Calculations of cable shape given in reference 1 show that a cable towing a body behind an airplane is fairly straight over a large part of its length. In any case, the analysis of the cable motion over a region of the cable should be approximately correct if the angle of attack is taken as that existing over the portion of the cable under consideration and if the radius of curvature is large compared to the wave length of the disturbances that are being considered.

To further simplify the notation let the trigonometric term $\cos \alpha_o + \dfrac{2 \sin^3\alpha_o}{\sin 2\alpha_o}$ equal γ. Also let

$$a^2 = \frac{T}{\mu}$$

and

$$\eta = \frac{KV_o^2}{2T}$$

After the steady-state terms are omitted and the remaining terms rearranged, equation (7) becomes

$$\frac{\partial^2 y}{\partial x^2} - 2\eta\frac{\partial y}{\partial x} - \frac{2\eta\gamma}{V_o}\frac{\partial y}{\partial t} = \frac{1}{a^2}\frac{\partial^2 y}{\partial t^2} \tag{8}$$

Equation (8) is the differential equation governing small transverse deviations of the cable from its equilibrium position. This equation differs from the equation for a cable with no aerodynamic forces by the addition of the second and third terms, which depend on aerodynamic effects. The equation without these two terms is the well-known wave equation. The solution of the wave equation shows that waves of arbitrary shape travel along the cable in either direction with the velocity a, and that the form and amplitude of the waves remain unchanged as they travel. (See, for example, reference 4.)

In order to study the propagation of waves along a cable in an air stream, it will be assumed that one point on the cable is forced to oscillate sinusoidally. The cable is assumed to extend from this point to infinity in either direction. A solution will be obtained for the resulting steady-state oscillation of the cable.

In a steady-state forced oscillation, all elements of the cable oscillate with the forcing frequency. For purposes of analysis, this oscillation is represented in the exponential form:

$$y = Ae^{i\omega t} \tag{9}$$

where A is a function of x. This expression gives the time dependence of the motion of the elements of the cable, hence

$$\frac{\partial y}{\partial t} = Ai\omega e^{i\omega t} = i\omega y$$

and

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 e^{i\omega t} = -\omega^2 y$$

If these expressions are substituted in equation (8), the equation becomes

$$\frac{\partial^2 y}{\partial x^2} - 2\eta\frac{\partial y}{\partial x} - \left(\frac{2\eta\gamma}{V_o} i\omega - \frac{\omega^2}{a^2}\right)y = 0 \tag{10}$$

The solution of this equation gives the position dependence of the motion of the cable elements. This solution has the form:

$$y = Ce^{\lambda x} \tag{11}$$

If this value is substituted in equation (10), the values of λ are obtained as roots of the equation

$$\lambda^2 - 2\eta\lambda - \left(\frac{2\eta\gamma}{V_o} i\omega - \frac{\omega^2}{a^2}\right) = 0$$

whence

$$\lambda = \eta \pm \sqrt{\eta^2 + \frac{2\eta\gamma}{V_o} i\omega - \frac{\omega^2}{a^2}}$$

This expression must be separated into its real and imaginary parts.

Let $\lambda = u + iv$. The following expressions are found for u and v:

$$u = \eta\left(1 + \frac{\gamma\omega}{V_o v}\right) \tag{12}$$

$$v = \pm\sqrt{\frac{1}{2}\left(\frac{\omega^2}{a^2} - \eta^2\right) \pm \sqrt{\left[\frac{1}{2}\left(\frac{\omega^2}{a^2} - \eta^2\right)\right]^2 + \frac{\gamma^2\omega^2\eta^2}{V_o^2}}} \tag{13}$$

By combining the solutions for the dependence of the cable position on the values of t (equation (9)) and x (equation (11)), the motion of the cable is represented by the expression

$$y = Ce^{(u+iv)x+i\omega t}$$

Or, if the real part of this expression is taken as the actual motion of the cable,

$$y = Ce^{ux} \cos\left(\omega t + vx\right) \tag{14}$$

DISCUSSION OF RESULTS

The solution for the motion of the cable, equation (14), indicates that trains of waves are traveling up and down the cable from the origin where the sinusoidal disturbance is applied. The waves traveling downwind correspond to a negative value of the term v, and those traveling upwind correspond to a positive value, as explained in reference 4.

The value of u, which determines the rate of increase or decrease in amplitude of the waves with distance from the origin, is different for the two sets of waves, because of the occurrence of the term v in the expression for u (formula (12)). When v is positive, corresponding to waves traveling upwind, both terms in the formula for u are positive. The amplitude of the oscillations, which is proportional to e^{ux}, always decreases with increasing distance upwind from the origin because this direction corresponds to negative values of x.

When v is negative, corresponding to waves traveling downwind, the terms in the formula for u have opposite signs. For small values of the airspeed, u will be negative, and the oscillation will therefore decrease with increasing distance downwind from the origin, because this direction corresponds to positive values of x. For high values of airspeed, however, u will be positive, and the oscillation will increase with increasing distance downwind from the origin.

The wave length of the disturbance propagated along the cable is

$$\Lambda = -\frac{2\pi}{v}$$

The period is

$$P = \frac{2\pi}{\omega}$$

Hence, the velocity of propagation of the waves is

$$U = \frac{\Lambda}{P} = -\frac{\omega}{v}$$

If U is substituted for $-\frac{\omega}{v}$ in the formula for u (formula (12)), the formula becomes

$$u = \eta\left(1 - \frac{\gamma U}{V_o}\right)$$

The value of γ is plotted as a function of α_o in figure 3. For small values of α_o the value of γ is very close to 1.0. In this case the value of u is approximately

$$u = \eta\left(1 - \frac{U}{V_o}\right)$$

The value of η is proportional to V_o^2. The dependence of the value of u on the velocity is therefore given by an expression of the form:

$$u \propto V_o^2 - UV_o$$

It may be seen that when $V_o = U$ (that is, when the airspeed is the same as the velocity of wave propagation) the value of u is zero. Under these conditions a disturbance travels downwind along the cable without change in amplitude. The solution therefore agrees with the physical concept that for small angles of attack the aerodynamic forces will have no effect when the wave moves at the same speed as the air. When the airspeed is less than the velocity of wave propagation, waves traveling downwind will be damped. When the airspeed is greater than the velocity of wave propagation, waves traveling downwind will increase in amplitude.

In the practical problem of an airplane towing a body by means of a cable, disturbances introduced at the airplane travel downwind along the cable until they reach the body. A part of these waves is then reflected upwind, the nature of the reflected wave depending on the amount of restraint provided by the body. The solution shows that the waves traveling upwind are well damped compared to the waves traveling downwind. The amplitude of the waves near the airplane, and in practice, over the greater part of the cable, may therefore be studied by considering only the waves moving downwind and neglecting the reflected waves or the nature of the restraint at the far end of the cable. The value of u is proportional to η (formula (12)). But

$$\eta = \frac{KV_o^2}{2T} = \frac{C_D \frac{\rho}{2} DV_o^2 \sin 2\alpha_o}{2T}$$

For small angles of attack, $\sin 2\alpha_o$ is proportional to α_o. Hence for small angles of attack, the value of u, which determines the rate of amplification of the oscillations, is proportional to α_o. A towed cable will ordinarily be pitched with respect to the air stream but not yawed. Any oscillations that occur would therefore be expected to take place in a vertical plane. In practice, the unstable motion of towed airspeed heads at high speeds has been observed to involve longitudinal motions predominantly.

The predicted lack of amplification of oscillations at zero angle of attack results from the assumption of infinitesimal oscillations and from the assumption that the aerodynamic force varies as $\sin^2\alpha$. Thus, the lift-curve slope for the cable is assumed to be zero at zero angle of attack. It may be expected that oscillations of finite amplitude will be amplified even at zero angle of attack because such oscillations will result in the development of aerodynamic forces on the cable elements.

Typical effects of airspeed on waves traveling along a towed cable have been calculated. The characteristics of the cable were taken as those of a cable used in conjunction with towed airspeed heads. These characteristics are as follows:

D, inch	3/8
μ, slug per foot	0.00165
T, pounds	30
C_D	1.2
ρ, slug per cubic foot	0.00238

In estimating the speed of propagation of the waves from the value of v (formula (13)) it was found that the term $\frac{\gamma^2\omega^2\eta^2}{V_o^2}$ was very small compared with other terms in the expression. The value of v then is almost exactly

$$v = \pm \sqrt{\frac{\omega^2}{a^2} - \eta^2}$$

Furthermore, for values of ω corresponding to frequencies greater than 3 cycles per second, the value of η^2 is small compared with $\frac{\omega^2}{a^2}$. For the assumed cable, then, and for frequencies of oscillation greater than 3 cycles per second, the value of v is approximately

$$v = \pm \frac{\omega}{a}$$

The velocity of the waves is therefore

$$U = - \frac{\omega}{v} = \pm a$$

Hence the velocity of propagation of the waves is nearly the same for the towed cable in the air stream as it would be for the cable in a vacuum. For the cable under consideration, a = 135 feet per second or 92 miles per hour.

At small angles of attack, the waves traveling down the cable will be amplified at airspeeds greater than 92 miles per hour. In order to show the extent of the amplification at various airspeeds and angles of attack, the amplitude of oscillation at a point on the cable 200 feet from the point of attachment to the airplane has been calculated for a unit amplitude of disturbance at the point of attachment. These results are shown in figure 4.

These curves show that for each angle of attack of the cable, the amplitude of the oscillation increases very rapidly as the speed is increased beyond a certain value. This rather abrupt onset of violent motion above a certain speed is apparently in accordance with the observed behavior of towed airspeed heads. The speed at which the oscillation is rapidly amplified is shown in figure 4 to decrease as the angle of attack of the cable is increased up to about 30°, but this speed increases slightly at 45°. The initial reduction in the speed for instability is caused by the increase of lift-curve slope of the cable with increase in angle of attack. At large angles of attack, however, the oscillation causes the cable elements to have a component of motion parallel to the wind direction. Damping forces are produced by this motion which cause the speed for instability to increase again.

An attempt has been made to determine whether the predictions of the theory are in reasonable agreement with the observed characteristics of towed airspeed heads. It was mentioned previously that the maximum speed at which a towed airspeed head of the type described has been successfully used depended on the type of airplane from which it is suspended, on the flap and power settings, and on the location from which it is lowered. On a large airplane in the flap-down condition, which would be expected to produce a large disturbance in the wake, the airspeed head has become unsteady at speeds as low as 135 miles per hour. In this case the motion was both lateral and vertical. In the case of an airplane in the clean condition in which the airspeed head was lowered from a position near the trailing edge of the wing, the head became unstable at 165 miles per hour. The maximum speed at which such an airspeed head has ever been known to remain stable is 275 miles per hour. When the instability occurs at high speed, it involves vertical motions predominantly.

Consideration of these facts leads to the belief that when the instability occurs at low speeds it may be due largely to the direct effect of disturbance in the wake; whereas when it occurs at high speeds, it is probably the result of a small oscillation caused by a disturbance acting on the cable near the airplane and amplified by the air forces acting on the cable. Inasmuch as the amplitude of the forcing motion is unknown in any particular case, a definite correlation between the theory and the observed characteristics of the airspeed head is not possible. The agreement of the theory with the observations may be shown to be reasonable, however, on the basis of the very rapid increase in amplification if the speed is increased beyond the values which have been reached in practice.

The curves shown in figure 4 are for a cable which is assumed to be straight. Usually, the cable will be curved. In order to estimate the rate of amplification of a wave traveling down a curved cable, it would be necessary to divide the cable into a series of segments each of which might be considered straight and to estimate the amplification taking place in each segment. This calculation may be made with the aid of a series of curves such as those shown in figure 5, which presents the cable length required for a disturbance to double in amplitude as a function of airspeed for various angles of attack of the cable. These curves were computed for the same cable characteristics as those assumed previously.

Calculations have been made to indicate the characteristics of the assumed cable and airspeed head at an airspeed of 270 miles per hour because this speed is close to the maximum speed at which this airspeed head has been successfully used. The equilibrium shape of such a cable 200 feet long when it is towing a 15-pound airspeed head has been estimated by means of a step-by-step solution. This analysis showed that at an airspeed of 270 miles per hour the cable is essentially straight over all except the final 30 feet of its length. The straight portion of the cable trails at an angle such that the cable weight is supported by the aerodynamic force on the cable. This angle varies inversely as the speed and is 5° at 270 miles per hour. The final 30 feet of the cable hooks down slightly due to the weight of the airspeed head. The tension in the cable increases from 16 pounds at the lower end to about 45 pounds at the airplane. The amplification of waves traveling down the cable may therefore be estimated quite well simply from the curve of figure 5 for a tension of 30 pounds and an angle of attack of 5°. This curve indicates that the amplitude would double each 52.3 feet, so that at a point 170 feet from the airplane, the amplitude would be about 9.5 times the forcing amplitude. This amplitude would probably be doubled again in the more steeply inclined portion at the end of the cable, though accurate estimations of the motion near the end have not been attempted, because reflected waves probably influence the motion in this region. It may be concluded, however, that an amplification of roughly 20 times the forcing amplitude would occur at this speed. The rapid increase in amplitude with further increase in speed makes it appear reasonable that 270 miles per hour is close to the maximum speed at which this type of airspeed head may be used.

Methods of increasing the speed at which instability of a towed body occurs may be examined on the basis of the theory. One method which has been suggested is to increase the weight of the towed body. This method has been proposed with the object of causing the cable to hang at a steeper angle so that it will be less influenced by the wake of the airplane. It has been shown, however, that very large increases in weight over that ordinarily used for airspeed heads would be required to influence appreciably the cable angle at the airplane, inasmuch as the major portion of the cable is nearly straight and hangs at an angle determined by its own weight rather than by the weight of the suspended body. Also the increased weight would increase the length of cable inclined steeply to the air stream, thereby increasing the amplification of waves. On the other hand, increasing the weight of the body would increase the cable tension, which would have the beneficial effect of increasing the speed of propagation of waves along the cable which is equal to $\sqrt{T/\mu}$. Finally, a heavier body would respond less violently to cable oscillations, so that larger cable motion might be possible without being objectionable. Despite these advantages, it does not appear that increasing the weight of the towed airspeed head within practical limits would greatly increase the speed at which it would remain steady in view of the rapid increase in cable instability with increasing speed.

A more promising means of increasing the speed at which violent motions of airspeed heads occur appears to be that of reducing the weight of the cable. This method would reduce the cable angle and increase the speed of propagation of waves. It would also reduce the response of the body to a given cable disturbance. A possible disadvantage of this method would be that of allowing the cable to be more influenced by the wake of the airplane. This problem might be avoided, however, by supporting the cable on a strut below the fuselage so that it would remain clear of the wake at all times.

The preceding discussion has been concerned mainly with towed airspeed heads. The applications of the theory to other towed devices, such as gliders and targets, is now discussed. A towed airspeed head is ordinarily a small, heavy, streamlined body which has small drag in comparison with its weight. As a result, this type of towed body introduces a tension force at the far end of the cable which is fairly independent of airspeed. On the other hand, gliders and targets have relatively large drag. These bodies cause a tension force in the cable which increases as the square of the airspeed. The speed of propagation of waves along the cable, given by $\sqrt{T/\mu}$, would therefore increase directly with the airspeed. It has been shown that the cable oscillations will not be amplified if the speed of the waves is greater than the airspeed. If the drag of the towed body is sufficiently large, therefore, the cable oscillations will be stable at any airspeed. The equivalent flat-plate area of the body required to make the velocity of the waves equal to the airspeed is

$$S = 1.56 \frac{\mu}{\rho}$$

In practice, a smaller drag might be sufficient, inasmuch as the cable oscillations are not amplified rapidly until the airspeed is considerably greater than the speed of wave propagation.

No cases are known in which cable oscillations have proved undesirable in the towing of gliders or targets. In these applications, the drag of the towed bodies has presumably been great enough to provide stability of the cable oscillations.

CONCLUDING REMARKS

It has been found from a theoretical study of the oscillations of a towed cable, that oscillations traveling downwind along the cable are amplified by aerodynamic forces when the airspeed is greater than the speed of propagation of waves along the cable. The oscillations are slightly damped when the airspeed is less than the speed of wave propagation. Waves traveling upwind along the cable are always damped, and the damping increases with increasing airspeed.

This theory provides a possible explanation for the violent motions of towed airspeed heads which appear above a certain speed. These oscillations are attributed to cable oscillations which originate near the airplane and are amplified by aerodynamic forces as they travel down the cable.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
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REFERENCES

1. Phillips, W. H.: Stability of a Body Stabilized by Fins and Suspended from an Airplane. NACA ARR No. L4D18, 1944.

2. Rayleigh, (Lord): The Theory of Sound. Second ed., vol. I, Macmillan & Co., Ltd. (London), 1896. (Reprinted 1929.) Sec. 148, pp. 232-234.

3. McLeod, A. R.: On the Action of Wind on Flexible Cables, with Applications to Cables Towed below Aeroplanes, and Balloon Cables. R & M No. 554, British A.C.A., 1918.

4. Frank, Nathaniel H.: Introduction to Mechanics and Heat. Second ed., McGraw-Hill Book Co., Inc., 1939, pp. 278-285.

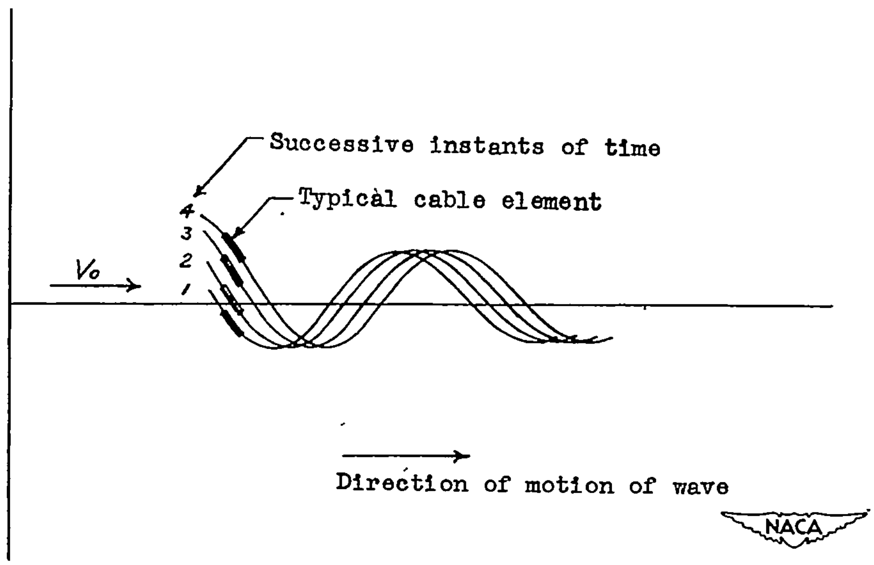

Figure 1.- Motion of cable element as wave travels down cable.

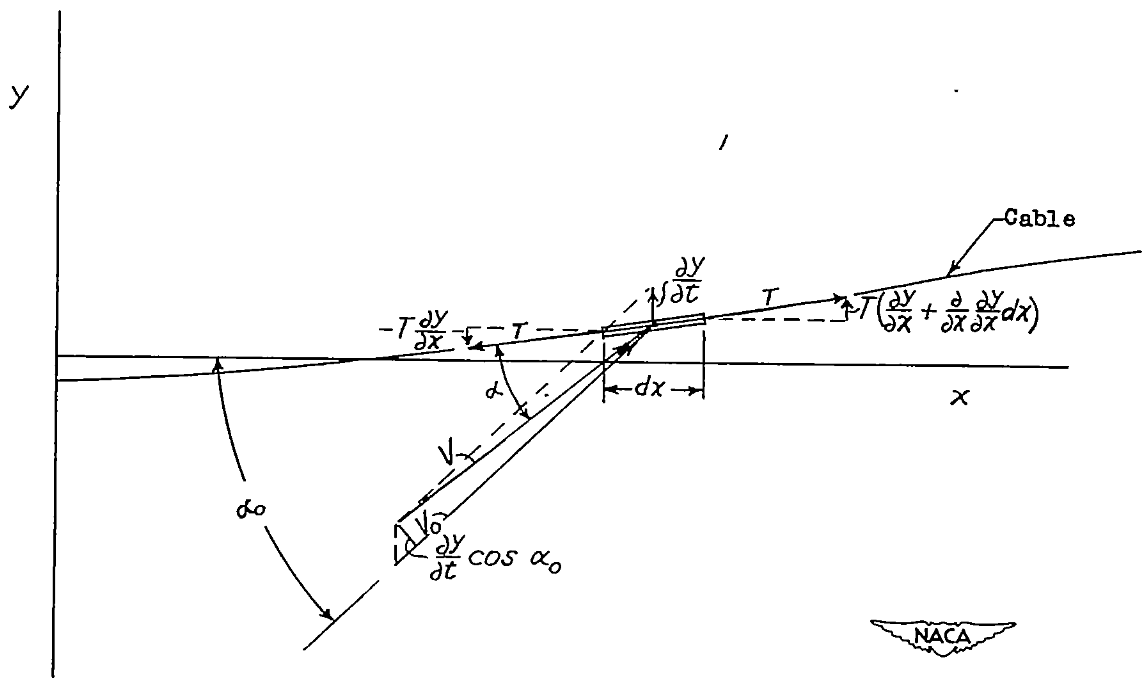

Figure 2.- Forces acting on an element of the cable.

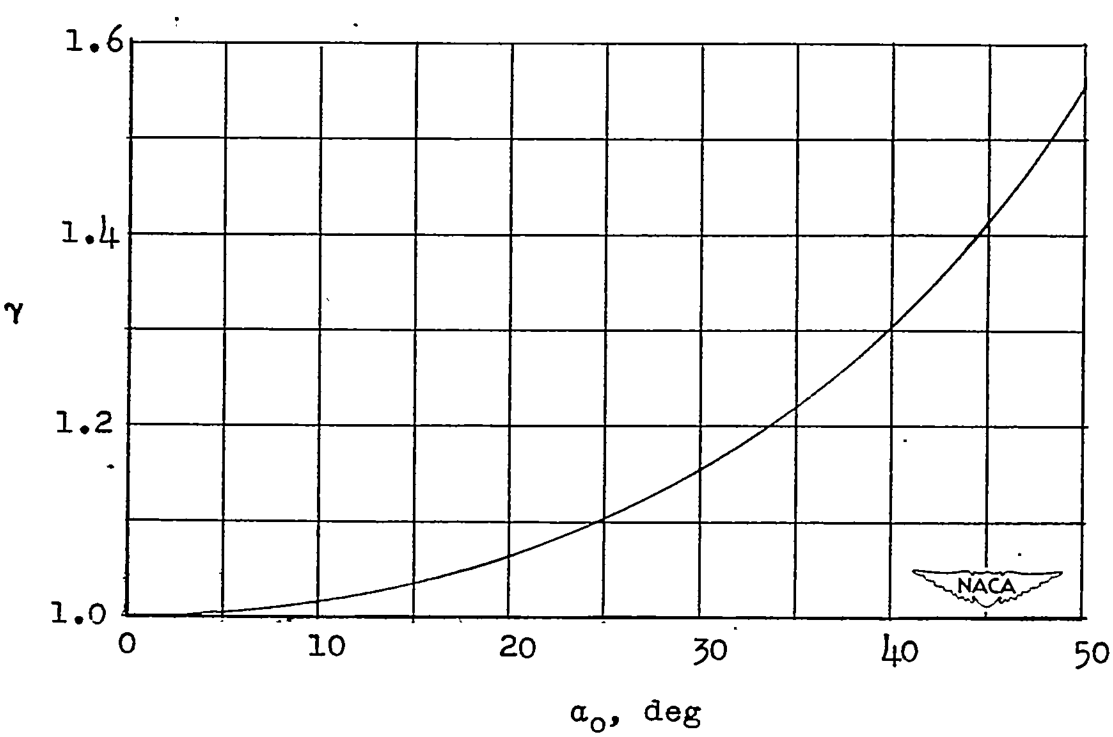

Figure 3.- Graph showing γ as a function of α_o.

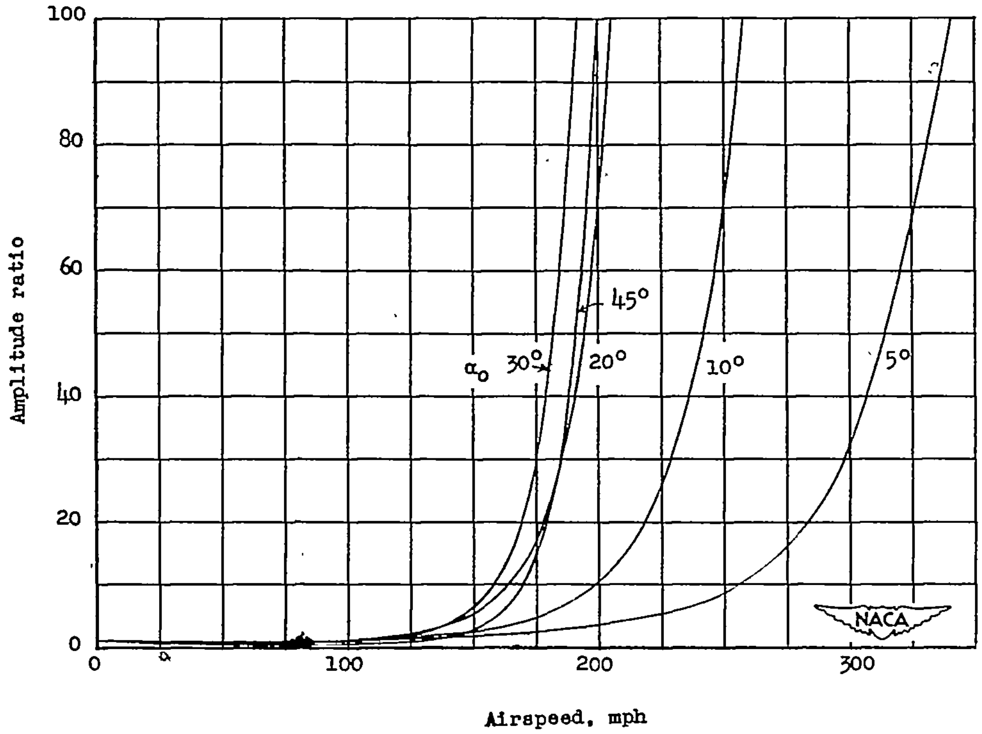

Figure 4.- Ratio of amplitude of oscillation at a point on cable 200 feet from airplane to forcing amplitude at airplane as a function of airspeed. Cable assumed straight.

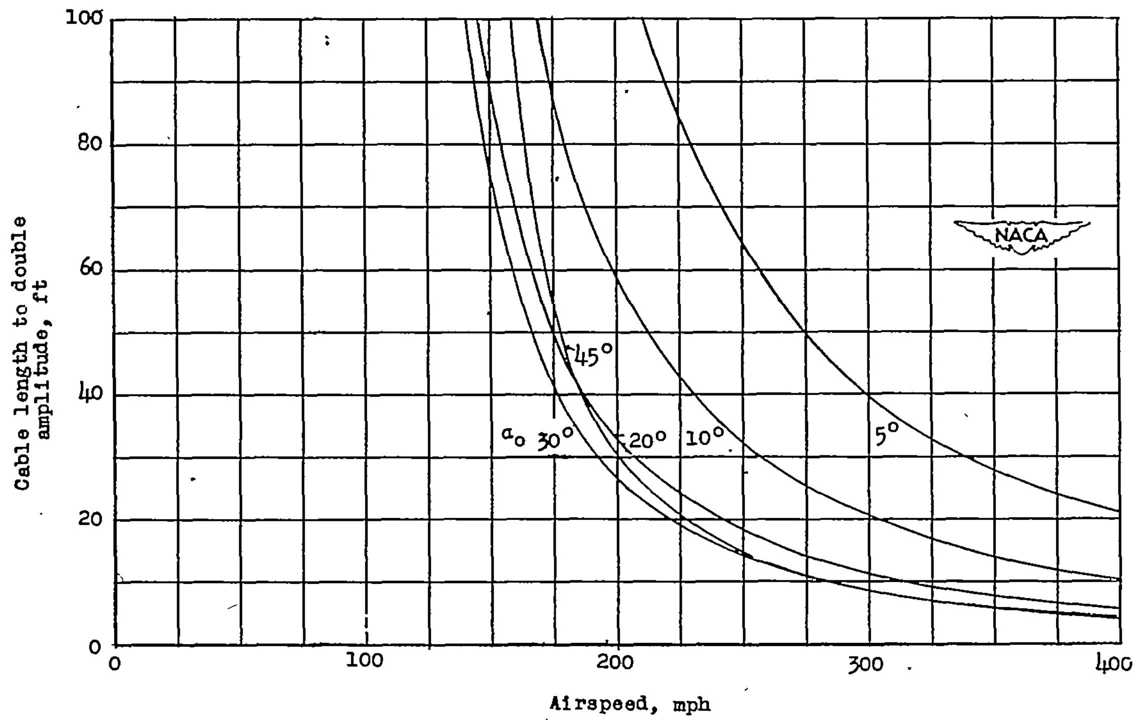

Figure 5.- Cable length in which oscillation doubles in amplitude as a function of airspeed.